v = velocity of flow
t = time
g = gravitational constant
μ = kinematic viscosity

In a previous paper,[1] Galletly asks for comments on his soap film paradox. The paradox arises when the forces on a part of the soap film are considered from the hypothesis that equilibrium exists. As approximately no flow of material takes place in a direction perpendicular to the film, there is no objection against the assumption of equilibrium in this direction. This leads to the equation

$$(\delta\varphi/\delta r) + (tg\varphi/r) = (h\sigma/2T) \tag{1}$$

However, in the direction tangential to the film, gravitational force can cause flow of material. A superficial observation of a soap film shows that if the film makes an angle with the horizontal plane, the thickness at the upper edge rapidly decreases.

In the equation of motion of the liquid the surface tension forces cancel. If viscosity forces are not taken into account, only gravitational force remains.

$$dv/dt = (\delta v/\delta t) + v\cos\varphi(\delta v/\delta r) = g\sin\varphi \tag{2}$$

The continuity equation gives

$$\cos\varphi(\delta/\delta r)(rhv) = r(\delta h/\delta t) \tag{3}$$

Equations (1–3) are partial differential equations for the unknown functions h,v,φ. They are not solved easily. Moreover, in reality, viscosity forces are to be taken in because, at the edges, adhesion forces prevent the film itself from flowing. The flow in the inner parts of the film therefore will be larger than in the outer parts. In this case, Eqs. (2) and (3) read

$$\frac{\delta v}{\delta t} + v\cos\varphi\frac{\delta v}{\delta r} = \mu\frac{\delta^2 v}{dx^2} + g\sin\varphi \tag{2a}$$

$$\cos\varphi\frac{\delta}{\delta r}\left(r\int_{-h/2}^{h/2} v\,dx\right) = r\frac{\delta h}{\delta t} \tag{3a}$$

Because of the constancy of the surface tension not only $v = 0$ but also $(\delta v/\delta x) = 0$ will prevail at the surfaces $x = \pm h/2$. The velocities will be smaller therefore and it will take more time before the film breaks down if the solution is of high viscosity.

Reference

[1] Galletly, R. A., "A note on a soap-film paradox," J. Aerospace Sci. 29, 1487–1488 (1962).

Note on Solution of a System of Three-Moment Equations

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In a recent paper, Wolff[1] gives a solution for a system of N-1 three-moment equations by expressing the inverted matrix of the system as a sum of N matrices, each N-1 by N-1 in size. By making use of recursion formulas given by Schechter,[2] it is possible to express the elements of the inverted matrix in terms of an invariant set of N numbers, a form that involves a small percentage of the calculation required in Ref. 1.

For the particular case of the three-moment equation given in Ref. 1,

$$M_{n-1} + 4M_n + M_{n+1} = -G_n \qquad n = 1,2,3,\ldots,N-1 \tag{1}$$

$$M_0 = 0 \qquad M_N = 0$$

Schechter's[2] recursion formulas reduce to

$$\begin{aligned} u_1 &= -G_1 \\ u_n &= -H_{n-1}G_n - u_{n-1} \end{aligned} \tag{2}$$
$$n = 2,3,\ldots,N-1$$

$$H_0 = 1 \qquad H_1 = 4 \qquad H_2 = 15$$
$$H_n = 4H_{n-1} - H_{n-2} \tag{3}$$
$$n = 2,3,\ldots,N-1$$

$$\begin{aligned} M_{N-1} &= u_{N-1}/H_{N-1} \\ M_{N-2} &= -G_{N-1} - 4M_{N-1} \\ M_{n-1} &= -G_n - 4M_n - M_{n+1} \end{aligned} \tag{4}$$
$$n = N-2, N-3,\ldots,2$$

Because of the simple form of the recursion formula in Eq. (2), it can be replaced by a simple sum to give u_{N-1}, the only value needed to start the recursion for the moments in Eq. (4):

$$\frac{u_{N-1}}{H_{N-1}} = M_{N-1} = \frac{1}{H_{N-1}}\sum_{i=1}^{N-1}(-1)^{N-i}H_{i-1}G_i \tag{5}$$

From the matrix form of Eq. (1)

$$QM = -G \tag{6}$$

the solution for M is

$$M = -Q^{-1}G \tag{7}$$

Examination of Eqs. (4) and (5) shows the elements q_{nm} of Q^{-1} on and to the left of the main diagonal have the form

$$q_{nm} = (-1)^{2N-n-m}\left(\frac{H_{N-n-1}H_{m-1}}{H_{N-1}}\right) \tag{8}$$
$$m = 1,2,3,\ldots,n$$
$$n = 1,2,3,\ldots,N-1$$

Since Q^{-1} is symmetrical, the elements to the right of the main diagonal are obtained by interchanging n and m in Eq (8).

Thus the solution for the moments is given either by Eqs. (5) and (4) or by Eqs. (8) and (7). Only the fixed set of numbers 1,4,15,56,209,780, . . . , given in Eq. (3) and the values of G_n, which represent the applied loads and temperatures acting between supports $n - 1$ and $n + 1$, are needed to make the calculations by either of the two methods. The amount of calculation by either procedure appears to be much smaller than that required in Ref. 1.

References

[1] Wolff, T., "Direct solution of the three-moments equation," AIAA J. 1, 718 (1963).
[2] Schechter, S., "Quasi-tridiagonal matrices and type-insensitive difference equations," Quart. Appl. Math. XVIII, 285–295 (1960).

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